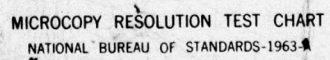END

DATE
FILMED

8-77

MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

OFFICE OF NAVAL RESEARCH

Contract N00014-75-C-0148

Project NR 064-436

Report No. UCB/AM-77-4

A Note on Invariance Under

Superposed Rigid Body Motions.

by

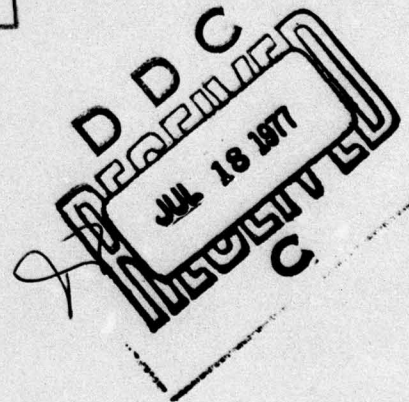A. E. Green P. M. Naghdi

Technical rept.,

Jun 1977

10 p.

D D C

JUL 18 1977

C

Department of Mechanical Engineering

University of California

Berkeley, California

A Note on Invariance Under

Superposed Rigid Body Motions

by

A. E. Green[†] and P. M. Naghdi[‡]

Consider a body with material points X and define a motion of the body by a sufficiently smooth vector function $\underset{\sim}{\chi}$ which assigns to each material point X the position $\underset{\sim}{x} = \underset{\sim}{\chi}(X,t)$ in the current configuration κ at time t. Under another motion, which differs from the given one only by a superposed rigid body motion, $\underset{\sim}{x}$ moves to $\underset{\sim}{x}^+ = \underset{\sim}{\chi}^+(X,t^+)$ in the configuration κ^+ at time $t^+ = t + a$, where a is a constant. It is well known that particles of a continuum, which at time t occupy the places $\underset{\sim}{x}$ and $\underset{\sim}{y}$, under superposed rigid body motions occupy at a different time t^+ the places $\underset{\sim}{x}^+$ and $\underset{\sim}{y}^+$ specified by

$$\underset{\sim}{x}^+ = \underset{\sim}{a} + \underset{\sim}{Q}\,\underset{\sim}{x} \quad , \quad \underset{\sim}{y}^+ = \underset{\sim}{a} + \underset{\sim}{Q}\,\underset{\sim}{y} \quad , \tag{1}$$

where $\underset{\sim}{a}$ is a vector function of t and $\underset{\sim}{Q}$ is a proper orthogonal tensor function of t. The vector $\underset{\sim}{a}$ in $(1)_{1,2}$ can be interpreted as a rigid body translation and $\underset{\sim}{Q}$ as a rotation tensor. The results (1) are a consequence of the definition that under superposed rigid body motions the magnitude of the relative displacement $|\underset{\sim}{y} - \underset{\sim}{x}|$ of two particles of the body remains unaltered for all pairs of particles. The relationships (1), in turn, induce certain transformations on various kinematical quantities represented as scalars, vectors or tensors and some of these transform according to formulas of the type

---

[†]Mathematical Institute, University of Oxford.
[‡]Department of Mechanical Engineering, University of California, Berkeley.

$$s^+ = s \quad , \quad \underset{\sim}{v}^+ = \underset{\sim}{Q}\,\underset{\sim}{v} \quad , \quad \underset{\sim}{S}^+ = \underset{\sim}{Q}\,\underset{\sim}{S}\,\underset{\sim}{Q}^T \quad , \tag{2}$$

where $s, \underset{\sim}{v}, \underset{\sim}{S}$ stand for a scalar, a vector and a second order tensor, respectively, and $\underset{\sim}{Q}^T$ denotes the transpose of $\underset{\sim}{Q}$. Of course, not all kinematical quantities transform according to (2) under superposed rigid body motions. In the current literature on continuum mechanics, functions and fields whose values are scalars, vectors and second order tensors and which obey transformations of the type (2) are often referred to as objective; and, for brevity, we also adopt this terminology here.[*]

In continuum mechanics, one also needs to consider the behavior of various constitutive response functions under superposed rigid body motions. Usually, it is assumed that these response functions are unaltered under superposed rigid body motions, apart from orientation in the case of vectors and tensors; and it is then stipulated that they satisfy conditions which have the same form as those in (2). The purpose of this note is to make these ideas more precise. An essential aspect of the development presented here, however, was given previously by Naghdi [1, pp. 484-486] for the special case of the stress vector.

Consider first a scalar field $\phi = \phi(\underset{\sim}{x}, t)$ defined over a three-dimensional region of Euclidean space $\mathcal{E}^3$ occupied by the body in the current configuration κ, and let $\phi^+ = \phi^+(\underset{\sim}{x}^+, t^+)$ be the corresponding scalar field defined over the region occupied by the body in the configuration κ^+ as a consequence of superposed rigid body motions. As noted above, the magnitude of the relative displacement vector $|\underset{\sim}{y} - \underset{\sim}{x}|$ remains unaltered under superposed rigid body motions and this objective character of a natural scalar magnitude suggests the following definition: The scalar field ϕ is said to be unaltered under superposed rigid body motions if

---

[*]Our use of the term objective, however, is different from the corresponding usage by many who appeal to the "principle of material frame-indifference" and thereby allow $\underset{\sim}{Q}$ to be an orthogonal tensor.

$$\phi^+ = \phi \; .$$
(3)

More generally, we assume that a relation of the type (3) holds if a scalar field depends on more than one spatial variable. Thus, for example, if $\psi = \psi(\underset{\sim}{x},\underset{\sim}{y},t)$ is objective, then $\psi^+(\underset{\sim}{x}^+,\underset{\sim}{y}^+,t^+) = \psi(\underset{\sim}{x},\underset{\sim}{y},t)$.

Consider next a vector field $\underset{\sim}{f} = \underset{\sim}{f}(\underset{\sim}{x},t)$ defined over a three-dimensional region of Euclidean space $\mathcal{E}^3$ occupied by the body in its current configuration κ, and let $\underset{\sim}{f}^+ = \underset{\sim}{f}^+(\underset{\sim}{x}^+,t^+)$ be the corresponding vector field defined over the region occupied by the body in the configuration κ^+ as a consequence of superposed rigid body motions (1). We say that under superposed rigid body motions a vector field $\underset{\sim}{f}$ is unaltered, apart from orientation, if

(i) the magnitude of $\underset{\sim}{f}(\underset{\sim}{x},t)$ is the same as the magnitude of $\underset{\sim}{f}^+(\underset{\sim}{x}^+,t^+)$; and

(ii) the magnitude of the angle between the vector $\underset{\sim}{f}(\underset{\sim}{x},t)$ and the relative displacement vector $\underset{\sim}{y}-\underset{\sim}{x}$ is the same as the magnitude of the angle between $\underset{\sim}{f}^+(\underset{\sim}{x}^+,t^+)$ and the relative vector $\underset{\sim}{y}^+-\underset{\sim}{x}^+$ when $\underset{\sim}{y}$ is any point of the current configuration different from $\underset{\sim}{x}$.

Let θ denote the angle between the vectors $\underset{\sim}{y}-\underset{\sim}{x}$ and $\underset{\sim}{f}$ and θ^+ the angle between $\underset{\sim}{y}^+-\underset{\sim}{x}^+$ and $\underset{\sim}{f}^+$. Then, since by (1)

$$\underset{\sim}{y}^+-\underset{\sim}{x}^+ = \underset{\sim}{Q}(\underset{\sim}{y}-\underset{\sim}{x}) \;\;,\;\; |\underset{\sim}{y}^+-\underset{\sim}{x}^+| = |\underset{\sim}{y}-\underset{\sim}{x}| \;\;,$$
(4)

it follows from (i) and (ii) above that $\underset{\sim}{f}$ must satisfy the inner product requirement

$$(\underset{\sim}{y}^+-\underset{\sim}{x}^+) \cdot \underset{\sim}{f}^+(\underset{\sim}{x}^+,t) = |\underset{\sim}{y}^+-\underset{\sim}{x}^+||\underset{\sim}{f}^+(\underset{\sim}{x}^+,t)|\cos\theta^+ = |\underset{\sim}{y}-\underset{\sim}{x}||\underset{\sim}{f}(\underset{\sim}{x},t)|\cos\theta = (\underset{\sim}{y}-\underset{\sim}{x}) \cdot \underset{\sim}{f}(\underset{\sim}{x},t)$$
(5)

for all points $\underset{\sim}{x},\underset{\sim}{y} \in \kappa$ and for all t in some closed time interval $[t_1,t_2]$. Since $\underset{\sim}{x},\underset{\sim}{y}$ are independent, we may differentiate (5) with respect to $\underset{\sim}{y}$ and also use (1) to obtain[†]

[†]The condition $(6)_2$ can also be deduced directly from $(5)_3$ without recourse to differentiation since (5) holds for all arbitrary vectors $\underset{\sim}{y}$ or $\underset{\sim}{y}-\underset{\sim}{x}$ (since $\underset{\sim}{x}$ is regarded as fixed) and since $\underset{\sim}{Q}^T\underset{\sim}{f}^+(\underset{\sim}{x}^+,t^+) - \underset{\sim}{f}(\underset{\sim}{x},t)$ is independent of $\underset{\sim}{y}$.

3.

$$Q^T \underset{\sim}{f}^+ = \underset{\sim}{f} \quad \text{or} \quad \underset{\sim}{f}^+ = Q \underset{\sim}{f} \ . \tag{6}$$

It can easily be verified that a vector function which transforms according to
(6) satisfies the conditions (i) and (ii). A vector field $\underset{\sim}{f}$ whose value obeys
the transformation $(6)_2$ under superposed rigid body motions may be referred to
as objective, in line with the terminology introduced above [following Eqs.
(2)]. We observe that the combination of (i) and (ii) is equivalent to the
requirement that the inner product $\underset{\sim}{f} \cdot (\underset{\sim}{y} - \underset{\sim}{x})$, designated as $\psi = \psi(\underset{\sim}{x}, \underset{\sim}{y}, t)$ say, is
an objective scalar:

$$\psi = \underset{\sim}{f} \cdot (\underset{\sim}{y} - \underset{\sim}{x}) \ , \quad \psi^+ = \underset{\sim}{f}^+ \cdot (\underset{\sim}{y}^+ - \underset{\sim}{x}^+) \ , \quad \psi^+ = \psi \ . \tag{7}$$

For reasons that will become apparent presently, we need to discuss further
the objectivity of a vector field such as $\underset{\sim}{f}$. Thus, let $\underset{\sim}{w} = \underset{\sim}{w}(\underset{\sim}{x}, t)$ be any objec-
tive vector field whose objective character could be tested by (7) after identifying
$\underset{\sim}{f}$ with $\underset{\sim}{w}$. Consider again any vector field $\underset{\sim}{g} = \underset{\sim}{g}(\underset{\sim}{x}, t)$ whose objectivity we wish
to investigate and suppose that $\underset{\sim}{g} \cdot \underset{\sim}{w}$ is known to be an objective scalar with $\underset{\sim}{g}$
independent of $\underset{\sim}{w}$:

$$\underset{\sim}{g}^+ \cdot \underset{\sim}{w}^+ = \underset{\sim}{g} \cdot \underset{\sim}{w} \ . \tag{8}$$

Since $\underset{\sim}{w}$ is objective, i.e.,

$$\underset{\sim}{w}^+ = Q \underset{\sim}{w} \ , \tag{9}$$

and since $Q^T \underset{\sim}{g}^+ - \underset{\sim}{g}$ is independent of $\underset{\sim}{w}$, from (8) we obtain

$$Q^T \underset{\sim}{g}^+ = \underset{\sim}{g} \ , \quad \underset{\sim}{g}^+ = Q \underset{\sim}{g} \ , \tag{10}$$

so that $\underset{\sim}{g}$ is an objective vector. The distinction between (7) and (8) is not
an idle point, since the latter provides a test for objectivity of a vector
field against any objective vector $\underset{\sim}{w}$ which is independent of $\underset{\sim}{g}$ and which is

4.

not necessarily a relative displacement vector.§

Now let $\underset{\sim}{F} = \underset{\sim}{F}(\underset{\sim}{x}, t)$ be a second order tensor field defined over the region of space occupied by the current configuration κ and let $\underset{\sim}{F}^+ = \underset{\sim}{F}^+(\underset{\sim}{x}^+, t^+)$ be the corresponding second order tensor field defined over the region occupied by the body in the configuration κ^+ as a result of superposed rigid body motions (1). We say that under superposed rigid body motions a tensor field $\underset{\sim}{F}$ is unaltered, apart from orientation, if

(iii) the magnitude of $\underset{\sim}{F}(\underset{\sim}{x}, t)$ is the same as the magnitude $\underset{\sim}{F}^+(\underset{\sim}{x}^+, t^+)$;

and

(iv) the magnitude of the angle θ between the tensor $\underset{\sim}{F}$ and the tensor $(\underset{\sim}{y}-\underset{\sim}{x}) \otimes (\underset{\sim}{z}-\underset{\sim}{x})$ is unaltered when $\underset{\sim}{y}$ and $\underset{\sim}{z}$ are any points of the current configuration different from $\underset{\sim}{x}$.

The condition (iv) requires, of course, the definition of an appropriate inner product. Such a definition is discussed below [see Eqs. (18)] in the more general context of tensors of order n.

From $(4)_1$ and a similar relation with $\underset{\sim}{y}, \underset{\sim}{y}^+$ replaced by $\underset{\sim}{z}, \underset{\sim}{z}^+$, we have

$$|(\underset{\sim}{y}^+ - \underset{\sim}{x}^+) \otimes (\underset{\sim}{z}^+ - \underset{\sim}{x}^+)| = |(\underset{\sim}{y}-\underset{\sim}{x}) \otimes (\underset{\sim}{z}-\underset{\sim}{x})| \tag{11}$$

and it follows from (iii) and (iv) that

$$\underset{\sim}{F}^+ \cdot (\underset{\sim}{y}^+ - \underset{\sim}{x}^+) \otimes (\underset{\sim}{z}^+ - \underset{\sim}{x}^+) = |\underset{\sim}{F}^+| \, |(\underset{\sim}{y}^+ - \underset{\sim}{x}^+) \otimes (\underset{\sim}{z}^+ - \underset{\sim}{x}^+)| \cos \theta^+$$

$$= |\underset{\sim}{F}| \, |(\underset{\sim}{y}-\underset{\sim}{x}) \otimes (\underset{\sim}{z}-\underset{\sim}{x})| \cos \theta$$

$$= \underset{\sim}{F} \cdot (\underset{\sim}{y}-\underset{\sim}{x}) \otimes (\underset{\sim}{z}-\underset{\sim}{x}) \ . \tag{12}$$

Since $\underset{\sim}{x}, \underset{\sim}{y}, \underset{\sim}{z}$ are independent, we may differentiate (12) successively with respect to $\underset{\sim}{y}$ and $\underset{\sim}{z}$ and use relations of the type (1) to obtain

_____

§It may be noted that an important application of (8) arises in the case of the scalar heat flux across any surface in the body which is assumed to be objective. Then, $\underset{\sim}{g}$ and $\underset{\sim}{y}$ are respectively identified with the heat flux vector and the outward unit normal to the surface, and we deduce that the heat flux vector is objective.

$$\underset{\sim}{F}^+ = \underset{\sim}{Q}\,\underset{\sim}{F}\,\underset{\sim}{Q}^T \; . \tag{13}$$

Alternatively, using relations of the form $(4)_1$, $(12)_3$ can be written as

$$(\underset{\sim}{Q}^T \underset{\sim}{F}^+ \underset{\sim}{Q} - \underset{\sim}{F}) \cdot (\underset{\sim}{y}-\underset{\sim}{x}) \otimes (\underset{\sim}{z}-\underset{\sim}{x}) = \underset{\sim}{0} \tag{14}$$

for all tensors $(\underset{\sim}{y}-\underset{\sim}{x}) \otimes (\underset{\sim}{z}-\underset{\sim}{x})$ independent of $\underset{\sim}{Q}^T \underset{\sim}{F}^+ \underset{\sim}{Q} - \underset{\sim}{F}$ and then (13) follows as before.

From $(4)_1$ and (13) we see that

$$\underset{\sim}{F}^+(\underset{\sim}{y}^+ - \underset{\sim}{x}^+) = \underset{\sim}{Q}\,\underset{\sim}{F}\,(\underset{\sim}{y}-\underset{\sim}{x}) \tag{15}$$

so that $\underset{\sim}{F}(\underset{\sim}{y}-\underset{\sim}{x})$ is an objective vector field. Thus, instead of using the definitions (iii) and (iv) for objectivity of a tensor field $\underset{\sim}{F}$ we could say that $\underset{\sim}{F}$ is objective provided the vector field $\underset{\sim}{F}(\underset{\sim}{y}-\underset{\sim}{x})$ is objective and we would then recover the result (13).

More generally, let $\underset{\sim}{w}$ be any objective vector field whose objective character could be tested by (7) after identifying $\underset{\sim}{f}$ with $\underset{\sim}{w}$, and consider any second order tensor field $\underset{\sim}{G} = \underset{\sim}{G}(\underset{\sim}{x},t)$ which is independent of $\underset{\sim}{w}$ and such that $\underset{\sim}{G}\,\underset{\sim}{w}$ is an objective vector field:

$$\underset{\sim}{G}^+ \underset{\sim}{w}^+ = \underset{\sim}{Q}\,\underset{\sim}{G}\,\underset{\sim}{w} \; . \tag{16}$$

In the same manner that the requirement (15) can lead to (13), from (16) we may obtain the result

$$\underset{\sim}{G}^+ = \underset{\sim}{Q}\,\underset{\sim}{G}\,\underset{\sim}{Q}^T \; . \tag{17}$$

Again it should be noted that the distinction between (15) and (16) is not an idle point: the latter permits a test for objectivity of a tensor field against

any objective vector field $\underset{\sim}{w}$ which is not necessarily a relative displacement.[*]

Finally, suppose $\underset{\sim}{V} = \underset{\sim}{V}(\underset{\sim}{x},t)$ is a tensor field of order n defined over the region of Euclidean space occupied by the body in the current configuration κ and let $\underset{\sim}{V}^+ = \underset{\sim}{V}^+(\underset{\sim}{x}^+,t^+)$ be the corresponding tensor field of order n defined over the region occupied by the body in the configuration κ^+ as a result of super-posed rigid body motions (1). Let $V_{i_1 i_2 \ldots i_n}$ denote the components of $\underset{\sim}{V}$ referred to an orthonormal basis $\underset{\sim}{e}_{i_1} \otimes \underset{\sim}{e}_{i_2} \otimes \cdots \otimes \underset{\sim}{e}_{i_n}$ for the 3^n-dimensional vector space $\mathcal{J}^n(\mathcal{E}^3)$ of tensors of order n defined over the n-fold product $\mathcal{E}^3 \times \mathcal{E}^3 \times \cdots \times \mathcal{E}^3$, and assume an appropriate inner product on $\mathcal{J}^n(\mathcal{E}^3)$, e.g.

$$\underset{\sim}{V} \cdot \underset{\sim}{W} = V_{i_1 i_2 \ldots i_n} W_{i_1 i_2 \ldots i_n} \, ,$$
$$|\underset{\sim}{V}| = (\underset{\sim}{V} \cdot \underset{\sim}{V})^{\frac{1}{2}} \, . \tag{18}$$

We say that under superposed rigid body motions a tensor field $\underset{\sim}{V}$ is unaltered, apart from orientation, if

(v) the magnitude of $\underset{\sim}{V}(\underset{\sim}{x},t)$ is the same as the magnitude of $\underset{\sim}{V}^+(\underset{\sim}{x}^+,t^+)$; and

(vi) the magnitude of the angle θ between the tensor $\underset{\sim}{V}$ and the tensor $(\underset{\sim}{y}_1 - \underset{\sim}{x}) \otimes (\underset{\sim}{y}_2 - \underset{\sim}{x}) \otimes \cdots \otimes (\underset{\sim}{y}_n - \underset{\sim}{x})$ is unaltered when $\underset{\sim}{y}_1, \ldots, \underset{\sim}{y}_n$ are any points of the current configuration different from $\underset{\sim}{x}$.

As in the case of (11), we have

$$|(\underset{\sim}{y}_1^+ - \underset{\sim}{x}^+) \otimes (\underset{\sim}{y}_2^+ - \underset{\sim}{x}) \otimes \cdots \otimes (\underset{\sim}{y}_n^+ - \underset{\sim}{x}^+)| = |(\underset{\sim}{y}_1 - \underset{\sim}{x}) \otimes (\underset{\sim}{y}_2 - \underset{\sim}{x}) \otimes \cdots \otimes (\underset{\sim}{y}_n - \underset{\sim}{x})| \tag{19}$$

and it follows from (v) and (vi) that

---

[*]An important application of (16) arises in the case of the stress vector across any surface in the body which is assumed to be objective. Then, $\underset{\sim}{G}$ and $\underset{\sim}{w}$ are respectively identified with the stress tensor and the outward unit normal to the surface and we deduce that the stress tensor is objective.

$$\underset{\sim}{v}^+ \cdot (\underset{\sim}{y}_1^+ - \underset{\sim}{x}^+) \otimes \cdots \otimes (\underset{\sim}{y}_n^+ - \underset{\sim}{x}^+) = |\underset{\sim}{v}^+| \ |(\underset{\sim}{y}_1^+ - \underset{\sim}{x}^+) \otimes \cdots \otimes (\underset{\sim}{y}_n^+ - \underset{\sim}{x}^+)| \cos \theta^+$$

$$= |\underset{\sim}{v}| \ |(\underset{\sim}{y}_1 - \underset{\sim}{x}) \otimes \cdots \otimes (\underset{\sim}{y}_n - \underset{\sim}{x})| \cos \theta$$

$$= \underset{\sim}{v} \cdot (\underset{\sim}{y}_1 - \underset{\sim}{x}) \otimes \cdots \otimes (\underset{\sim}{y}_n - \underset{\sim}{x}) \ . \tag{20}$$

Then, since $\underset{\sim}{y}_1, \ldots, \underset{\sim}{y}_n$ are independent, we may differentiate (20) successively with respect to $\underset{\sim}{y}_1, \ldots, \underset{\sim}{y}_n$ and use relations of the type (1) to obtain

$$\underset{\sim}{v}^+ = v^+_{i_1 i_2 \ldots i_n} \underset{\sim}{e}_{i_1} \otimes \underset{\sim}{e}_{i_2} \otimes \cdots \otimes \underset{\sim}{e}_{i_n} = Q_{i_1 j_1} Q_{i_2 j_2} \cdots Q_{i_n j_n} V_{j_1 j_2 \ldots j_n} \underset{\sim}{e}_{i_1} \otimes \underset{\sim}{e}_{i_2} \otimes \cdots \otimes \underset{\sim}{e}_{i_n} \ , \tag{21}$$

where $Q_{i_1 i_2}$ are the components of the proper orthogonal tensor $\underset{\sim}{Q}$ referred to the orthonormal basis $\underset{\sim}{e}_{i_1} \otimes \underset{\sim}{e}_{i_2}$ for the $3^2 = 9$-dimensional vector space $\mathcal{J}^2(\mathcal{E}^3)$ of second order tensors defined over the product space $\mathcal{E}^3 \times \mathcal{E}^3$. Further immediate generalizations of the result (21), for example similar to that discussed above for second order tensors, are possible but we do not pursue the matter further.

Reference

1. Naghdi, P. M., The theory of shells and plates. In S. Flügge's Handbuch der Physik, Vol. VI/a (edited by C. Truesdell), pp. 425-640, Springer-Verlag, 1972.

SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)*

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER UCB/AM-77-4	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE *(and Subtitle)* A Note on Invariance Under Superposed Rigid Body Motions		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(*s*) A. E. Green and P. M. Naghdi		8. CONTRACT OR GRANT NUMBER(*s*) N00014-75-C-0148
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Mechanical Engineering University of California Berkeley, California 94720		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 064-436
11. CONTROLLING OFFICE NAME AND ADDRESS Structural Mechanics Program Office of Naval Research Arlington, Virginia 22217		12. REPORT DATE June 1977
		13. NUMBER OF PAGES 8
14. MONITORING AGENCY NAME & ADDRESS(*If different from Controlling Office*)		15. SECURITY CLASS. *(of this report)*
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

Invariance conditions, superposed rigid body motions, constitutive response functions, scalars, vectors, tensors.

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

An important aspect in the determination of the material response functions, which may be represented as scalars, vectors or tensors, involves the use of appropriate invariance requirements under superposed rigid body motions. This note is concerned with these invariance requirements in which the relevant ideas are better motivated and are made more precise.

DD $_{1 \text{ JAN } 73}^{\text{FORM}}$ 1473   EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601 |